Lesson 8

41. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x$. *Ans.* e^2. 42. $\lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)^x$. *Ans.* $\frac{1}{e}$. 43. $\lim\limits_{x\to\infty}\left(\frac{x}{1+x}\right)^x$. *Ans.* $\frac{1}{e}$. 44. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}$ *Ans.* e. 45. $\lim\limits_{n\to\infty}\{n[\ln(n+1)-\ln n]\}$. *Ans.* 1.

46. $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos x)^{3\sec x}$. *Ans.* e^3. 47. $\lim\limits_{x\to 0}\frac{\ln(1+\alpha x)}{x}$. *Ans.* α. 48. $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$. *Ans.* e.

49. $\lim\limits_{x\to 0}(1+3\tan^2 x)^{\cot^2 x}$. *Ans.* e^3. 50. $\lim\limits_{m\to\infty}\left(\cos\frac{x}{m}\right)^m$. *Ans.* 1.

51. $\lim\limits_{\alpha\to\infty}\frac{\ln(1+e^\alpha)}{\alpha}$. *Ans.* 1 as $\alpha\longrightarrow+\infty$, 0 as $\alpha\longrightarrow-\infty$. 52. $\lim\limits_{x\to 0}\frac{\sin\alpha x}{\sin\beta x}$. *Ans.* $\frac{\alpha}{\beta}$. 53. $\lim\limits_{x\to\infty}\frac{a^x-1}{x}$ $(a>1)$. *Ans.* $+\infty$ as $x\longrightarrow+\infty$, 0 as $x\longrightarrow-\infty$.

54. $\lim\limits_{n\to\infty} n\left[a^{\frac{1}{n}}-1\right]$. *Ans.* $\ln a$. 55. $\lim\limits_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{x}$. *Ans.* $\alpha-\beta$.

56. $\lim\limits_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{\sin\alpha x-\sin\beta x}$. *Ans.* 1.

Determine the points of discontinuity of the functions:

57. $y=\frac{x-1}{x(x+1)(x^2-4)}$. *Ans.* Discontinuities at $x=-2, -1, 0, 2$. 58. $y=\tan\frac{1}{x}$. *Ans.* Discontinuities at $x=0$ and $x=\pm\frac{2}{\pi}, \pm\frac{2}{3\pi}, \dots, \pm\frac{2}{(2n+1)\pi}, \dots$.

59. Find the points of discontinuity of the function $y=1+2^{\frac{1}{x}}$ and construct the graph of this function. *Ans.* Discontinuity at $x=0$ ($y\longrightarrow+\infty$ as $x\longrightarrow 0+0$, $y\longrightarrow 1$ as $x\longrightarrow 0-0$).

60. From among the following infinitesimals (as $x\longrightarrow 0$): x^2, $\sqrt{x(1-x)}$, $\sin 3x$, $2x\cos x\sqrt[3]{\tan^2 x}$, xe^{2x}, select infinitesimals of the same order as x, and also of higher and lower order than x. *Ans.* Infinitesimals of the same order as x are $\sin 3x$ and xe^{2x}; infinitesimals of higher order than x, x^2 and $2x\cos x\sqrt[3]{\tan^2 x}$, infinitesimals of lower order than x, $\sqrt{x(1-x)}$.

61. Choose from among the same infinitesimals (as $x\longrightarrow 0$) such that are equivalent to the infinitesimal x: $2\sin x$, $\frac{1}{2}\tan 2x$, $x-3x^2$, $\sqrt{2x^2+x^3}$, $\ln(1+x)$, x^3+3x^4. *Ans.* $\frac{1}{2}\tan 2x$, $x-3x^2$, $\ln(1+x)$.

62. Check to see that as $x\longrightarrow 1$, the infinitesimals $1-x$ and $1-\sqrt[3]{x}$ are of the same order. Are they equivalent? *Ans.* $\lim\limits_{x\to 1}\frac{1-x}{1-\sqrt[3]{x}}=3$; hence, these infinitesimals are of the same order, but they are not equivalent.

Even examples must be solved in class, odd examples must be solved at home