Lesson 31 (even problems must be solved in class, odd examples must be solved at home)

1. Find $(3+5i)(4-i)$. *Ans.* $17+17i$. **2.** Find $(6+11i)(7+3i)$. *Ans.* $9+95i$. **3.** Find $\frac{3-i}{4+5i}$. *Ans.* $\frac{7}{41}-\frac{19}{41}i$. **4.** Find $(4-7i)^3$. *Ans.* $-524+7i$. **5.** Find $\sqrt{i}$. *Ans.* $\pm\frac{1+i}{\sqrt{2}}$. **6.** Find $\sqrt{-5-12i}$. *Ans.* $\pm(2-3i)$. **7.** Reduce the following expressions to trigonometric form: (a) $1+i$. *Ans.* $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, (b) $1-i$. *Ans.* $\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$. **8.** Find $\sqrt[3]{i}$. *Ans.* $\frac{i+\sqrt{3}}{2}$, $-i$, $\frac{i-\sqrt{3}}{2}$. **9.** Express the following expressions in terms of powers of $\sin x$ and $\cos x$: $\sin 2x$, $\cos 2x$, $\sin 4x$, $\cos 4x$, $\sin 5x$, $\cos 5x$. **10.** Express the following in terms of the sine and cosine of multiple arcs: $\cos^2 x$, $\cos^3 x$, $\cos^4 x$, $\cos^5 x$, $\cos^6 x$; $\sin^2 x$, $\sin^3 x$, $\sin^4 x$, $\sin^5 x$. **11.** Divide $f(x)=x^3-4x^2+8x-1$ by $x+4$. *Ans.* $f(x)=(x+4)(x^2-8x+40)-161$, that is, the quotient is equal to $x^2-8x+40$; and the remainder is $f(-4)=-161$. **12.** Divide $f(x)=x^4+12x^3+54x^2+108x+81$ by $x+3$. *Ans.* $f(x)=(x+3)(x^3+9x^2+27x+27)$. **13.** Divide $f(x)=x^7-1$ by $x-1$. *Ans.* $f(x)=(x-1)(x^6+x^5+x^4+x^3+x^2+x+1)$.

Factor the following polynomials into factors with real coefficients:

14. $f(x)=x^4-1$. *Ans.* $f(x)=(x-1)(x+1)(x^2+1)$. **15.** $f(x)=x^2-x-2$. *Ans.* $f(x)=(x-2)(x+1)$. **16.** $f(x)=x^3+1$. *Ans.* $f(x)=(x+1)(x^2-x+1)$.

17. Experiment yielded the following values of y as a function of x:

$$y_1=4 \quad \text{for} \quad x_1=0$$
$$y_2=6 \quad \text{for} \quad x_2=1$$
$$y_3=10 \quad \text{for} \quad x_3=2$$

Approximate the function by a second-degree polynomial. *Ans.* x^2+x+4.

18. Find a polynomial of degree four that takes on the values 2, 1, -1, 5, 0 for $x=1, 2, 3, 4, 5$, respectively. *Ans.* $-\frac{7}{6}x^4+\frac{79}{6}x^3-\frac{151}{3}x^2+\frac{226}{3}x-35$.

19. Find a polynomial of the lowest possible degree that takes on the values 3, 7, 9, 19 for $x=2, 4, 5, 10$, respectively. *Ans.* $2x-1$.

20. Find Bernstein polynomials of degree 1, 2, 3 and 4 for the function $y=\sin \pi x$ on the interval $[0, 1]$. *Ans.* $B_1(x)=0$, $B_2(x)=2x(1-x)$, $B_3(x)=\frac{3\sqrt{3}}{2}x(1-x)$, $B_4(x)=2x(1-x)\left[(2\sqrt{2}-3)x^2-(2\sqrt{2}-3)x+\sqrt{2}\right]$.