Lesson 24 (even problems must be solved in class, odd examples must be solved at home)

Find the extrema of the functions:

1. $y=x^2-2x+3$. *Ans.* $y_{\min}=2$ at $x=1$. **2.** $y=\frac{x^3}{3}-2x^2+3x+1$. *Ans.* $y_{\max}=\frac{7}{3}$ at $x=1$, $y_{\min}=1$ at $x=3$. **3.** $y=x^3-9x^2+15x+3$. *Ans.* $y_{\max}=10$ at $x=1$, $y_{\min}=-22$ at $x=5$. **4.** $y=-x^4+2x^2$. *Ans.* $y_{\max}=1$ at $x=\pm 1$, $y_{\min}=0$ at $x=0$. **5.** $y=x^4-8x^2+2$. *Ans.* $y_{\max}=2$ at $x=0$, $y_{\min}=-14$ at $x=\pm 2$. **6.** $y=3x^5-125x^3+2160x$. *Ans.* Maximum at $x=-4$ and $x=3$, minimum at $x=-3$ and $x=4$. **7.** $y=2-(x-1)^{\frac{2}{3}}$. *Ans.* $y_{\max}=2$ at $x=1$. **8.** $y=3-2(x+1)^{\frac{1}{3}}$. *Ans.* There is neither maximum nor minimum. **9.** $y=\frac{x^2-3x+2}{x^2+3x+2}$. *Ans.* Minimum at $x=\sqrt{2}$, maximum at $x=-\sqrt{2}$. **10.** $y=\frac{(x-2)(3-x)}{x^2}$. *Ans.* Maximum at $x=\frac{12}{5}$. **11.** $y=2e^x+e^{-x}$. *Ans.* Minimum at $x=-\frac{\ln 2}{2}$. **12.** $y=\frac{x}{\ln x}$. *Ans.* $y_{\min}=e$ at $x=e$. **13.** $y=\cos x+\sin x\left(-\frac{\pi}{2}\leqslant x\leqslant\frac{\pi}{2}\right)$. *Ans.* $y_{\max}=\sqrt{2}$ at $x=\frac{\pi}{4}$. **14.** $y=\sin 2x-x\left(-\frac{\pi}{2}\leqslant x\leqslant\frac{\pi}{2}\right)$. *Ans.* Maximum at $x=\frac{\pi}{6}$, minimum at $x=-\frac{\pi}{6}$. **15.** $y=x+\tan x$. *Ans.* There is neither maximum nor minimum. **16.** $y=e^x\sin x$. *Ans.* Minimum at $x=2k\pi-\frac{\pi}{4}$, maximum at $x=2k\pi+\frac{3}{4}\pi$. **17.** $y=x^4-2x^2+2$. *Ans.* Maximum at $x=0$, two minima when $x=-1$ and when $x=1$. **18.** $y=(x-2)^3(2x+1)$. *Ans.* $y_{\min}\approx -8.24$ when $x=\frac{1}{8}$. **19.** $y=x+\frac{1}{x}$. *Ans.* Minimum when $x=1$, maximum when $x=-1$. **20.** $y=x^2(a-x)^2$. *Ans.* $y_{\max}=\frac{a^4}{16}$ when $x=\frac{a}{2}$, $y_{\min}=0$ when $x=0$ and when $x=a$. **21.** $y=\frac{a^2}{x}+\frac{b^2}{a-x}$. *Ans.* Maximum when $x=\frac{a^2}{a-b}$, minimum when $x=\frac{a^2}{a+b}$. **22.** $y=x+\sqrt{1-x}$. *Ans.* $y_{\max}=\frac{5}{4}$ when $x=\frac{3}{4}$, $y_{\min}=1$ when $x=1$. **23.** $y=x\sqrt{1-x}$ $(x\leqslant 1)$. *Ans.* $y_{\max}=\frac{2}{3}\sqrt{\frac{1}{3}}$ when $x=\frac{2}{3}$. **24.** $y=\frac{x}{1+x^2}$. *Ans.* Minimum when $x=-1$, maximum when $x=1$. **25.** $y=x\ln x$. *Ans.* Minimum when $x=\frac{1}{e}$. **26.** $y=x\ln^2 x$. *Ans.* $y_{\max}=4e^{-2}$ at $x=e^{-2}$, $y_{\min}=0$ at $x=1$. **27.** $y=\ln x-\arctan x$. *Ans.* The function increases. **28.** $y=\sin 3x-3\sin x$. *Ans.* Minimum when $x=\frac{\pi}{2}$, maximum when $x=\frac{3\pi}{2}$. **29.** $y=2x+\arctan x$. *Ans.* No extrema.. **30.** $y=\sin x\cos^2 x$. *Ans.* Minimum when $x=\frac{\pi}{2}$, two maxima

when $x=\arccos\sqrt{\frac{2}{3}}$ and when $x=\arccos\left(-\sqrt{\frac{2}{3}}\right)$. **31.** $y=\arcsin(\sin x)$. *Ans.* Maximum when $x=\frac{(4m+1)\pi}{2}$, minimum when $x=\frac{(4m+3)\pi}{2}$.

Find the maximum and minimum values of the function on the indicated intervals:

32. $y=-3x^4+6x^2-1$ $(-2\leqslant x\leqslant 2)$. *Ans.* Maximum $y=2$ at $x=\pm 1$, minimum $y=-25$ at $x=\pm 2$. **33.** $y=\frac{x^3}{3}-2x^2+3x+1$ $(-1\leqslant x\leqslant 5)$. *Ans.* Maximum value $y=\frac{23}{3}$ at $x=5$, minimum value $y=-\frac{13}{3}$ at $x=-1$. **34.** $y=\frac{x-1}{x+1}$ $(0\leqslant x\leqslant 4)$. *Ans.* Maximum value $y=\frac{3}{5}$ at $x=4$, minimum value $y=-1$ at $x=0$. **35.** $y=\sin 2x-x\left(-\frac{\pi}{2}\leqslant x\leqslant\frac{\pi}{2}\right)$. *Ans.* Maximum value $y=\frac{\pi}{2}$ at $x=-\frac{\pi}{2}$, minimum value $y=-\frac{\pi}{2}$ at $x=\frac{\pi}{2}$.