Lesson 22 (even problems must be solved in class, odd examples must be solved at home)

53. Expand, in powers of $x-2$, the polynomial $x^4-5x^3+5x^2+x+2$. *Ans.* $-7(x-2)-(x-2)^2+3(x-2)^3+(x-2)^4$. **54.** Expand, in powers of $x+1$, the polynomial $x^5+2x^4-x^2+x+1$. *Ans.* $(x+1)^2+2(x+1)^3-3(x+1)^4+(x+1)^5$. **55.** Write Taylor's formula for the function $y=\sqrt{x}$ when $a=1$, $n=3$. *Ans.* $\sqrt{x}=1+\frac{x-1}{1}\cdot\frac{1}{2}-\frac{(x-1)^2}{1\cdot 2}\cdot\frac{1}{4}+$ $+\frac{(x-1)^3}{1\cdot 2\cdot 3}\cdot\frac{3}{8}-\frac{(x-1)^4}{4!}\cdot\frac{15}{16}\cdot[1+\theta(x-1)]^{-\frac{7}{2}}$, $0<\theta<1$. **56.** Write the Maclaurin formula for the function $y=\sqrt{1+x}$ when $n=2$. *Ans.* $\sqrt{1+x}=1+$ $+\frac{1}{2}x-\frac{1}{8}x^2+\frac{x^3}{16(1+\theta x)^{\frac{5}{2}}}$, $0<\theta<1$. **57.** Using the results of the preceding exercise, estimate the error of the approximate equation $\sqrt{1+x}\approx$ $\approx 1+\frac{1}{2}x-\frac{1}{8}x^2$ when $x=0.2$. *Ans.* Less than $\frac{1}{2\cdot 10^3}$.

Determine the origin of the approximate equations for small values of x and estimate the errors of these equations: **58.** $\ln\cos x\approx-\frac{x^2}{2}-\frac{x^4}{12}$. **59.** $\tan x\approx x+\frac{x^3}{3}+\frac{2x^5}{15}$. **60.** $\arcsin x\approx x+\frac{x^3}{6}$. **61.** $\arctan x\approx x-\frac{x^3}{3}$. **62.** $\frac{e^x+e^{-x}}{2}\approx 1+\frac{x^2}{2}+\frac{x^4}{24}$. **63.** $\ln(x+\sqrt{1-x^2})\approx x-x^2+\frac{5x^3}{6}$.

Using Taylor's formula, compute the limits of the following expressions: **64.** $\lim\limits_{x\to 0}\frac{x-\sin x}{e^x-1-x-\frac{x^2}{2}}$. *Ans.* 1. **65.** $\lim\limits_{x\to 0}\frac{\ln^2(1+x)-\sin^2 x}{1-e^{-x^2}}$. *Ans.* 0. **66.** $\lim\limits_{x\to 0}\frac{2(\tan x-\sin x)-x^3}{x^5}$. *Ans.* $\frac{1}{4}$. **67.** $\lim\limits_{x\to 0}\left[x-x^2\ln\left(1+\frac{1}{x}\right)\right]$. *Ans.* 0. **68.** $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-\frac{\cot x}{x}\right)$. *Ans.* $\frac{1}{3}$. **69.** $\lim\limits_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$. *Ans.* $\frac{2}{3}$.