Lesson 21 (even problems must be solved in class, odd examples must be solved at home)

Evaluate the following limits:

18. $\lim\limits_{x\to 1}\dfrac{x-1}{x^n-1}$. *Ans.* $\dfrac{1}{n}$. **19.** $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{\sin x}$. *Ans.* 2. **20.** $\lim\limits_{x\to 0}\dfrac{\tan x-x}{x-\sin x}$. *Ans.* 2.

21. $\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{\cos x-1}$. *Ans.* -2. **22.** $\lim\limits_{x\to 0}\dfrac{\sin x}{\sqrt{1-\cos x}}$. *Ans.* There is no limit ($\sqrt{2}$ as $x\to+0$, $-\sqrt{2}$ as $x\to-0$). **23.** $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\ln\sin x}{(\pi-2x)^2}$. *Ans.* $-\dfrac{1}{8}$.

24. $\lim\limits_{x\to 0}\dfrac{a^x-b^x}{x}$. *Ans.* $\ln\dfrac{a}{b}$. **25.** $\lim\limits_{x\to 0}\dfrac{x-\arcsin x}{\sin^3 x}$. *Ans.* $-\dfrac{1}{6}$.

26. $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$. *Ans.* $\cos a$. **27.** $\lim\limits_{y\to 0}\dfrac{e^y+\sin y-1}{\ln(1+y)}$. *Ans.* 2.

28. $\lim\limits_{x\to 0}\dfrac{e^x\sin x-x}{3x^2+x^5}$. *Ans.* $\dfrac{1}{3}$. **29.** $\lim\limits_{x\to\infty}\dfrac{3x-1}{2x+5}$. *Ans.* $\dfrac{3}{2}$. **30.** $\lim\limits_{x\to\infty}\dfrac{\ln x}{x^n}$ (where $n>0$). *Ans.* 0. **31.** $\lim\limits_{x\to\infty}\dfrac{\ln\left(1+\frac{1}{x}\right)}{\arctan x}$. *Ans.* 1. **32.** $\lim\limits_{x\to\infty}\dfrac{\ln\frac{x+1}{x}}{\ln\frac{x-1}{x}}$. *Ans.* -1.

33. $\lim\limits_{y\to+\infty}\dfrac{y}{e^{ay}}$. *Ans.* 0 for $a>0$, ∞ for $a\leqslant 0$. **34.** $\lim\limits_{x\to+\infty}\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$. *Ans.* 1. **35.** $\lim\limits_{x\to 0}\dfrac{\ln\sin 3x}{\ln\sin x}$. *Ans.* 1. **36.** $\lim\limits_{x\to 0}\dfrac{\ln\tan 7x}{\ln\tan 2x}$. *Ans.* 1.

37. $\lim\limits_{x\to 1}\dfrac{\ln(x-1)-x}{\tan\frac{\pi}{2x}}$. *Ans.* 0. **38.** $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi x}{2}$. *Ans.* $\dfrac{2}{\pi}$.

39. $\lim\limits_{x\to 1}\left[\dfrac{2}{x^2-1}-\dfrac{1}{x-1}\right]$. *Ans.* $-\dfrac{1}{2}$. **40.** $\lim\limits_{x\to 1}\left[\dfrac{1}{\ln x}-\dfrac{x}{\ln x}\right]$. *Ans.* -1.

41. $\lim\limits_{\varphi\to\frac{\pi}{2}}(\sec\varphi-\tan\varphi)$. *Ans.* 0. **42.** $\lim\limits_{x\to 1}\left[\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right]$. *Ans.* $\dfrac{1}{2}$. **43.** $\lim\limits_{x\to 0}x\cot 2x$. *Ans.* $\dfrac{1}{2}$. **44.** $\lim\limits_{x\to 0}x^2e^{\frac{1}{x^2}}$. *Ans.* ∞. **45.** $\lim\limits_{x\to 1}x^{\frac{1}{1-x}}$ *Ans.* $\dfrac{1}{e}$. **46.** $\lim\limits_{t\to\infty}\sqrt[t]{t^2}$. *Ans.* 1.

47. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}\right)^{\tan x}$. *Ans.* 1. **48.** $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}\right)^x$. *Ans.* e^a. **49.** $\lim\limits_{x\to 0}(\cot x)^{\frac{1}{\ln x}}$. *Ans.* $\dfrac{1}{e}$. **50.** $\lim\limits_{x\to\frac{\pi}{2}}(\cos x)^{\frac{\pi}{2}-x}$. *Ans.* 1. **51.** $\lim\limits_{\varphi\to 0}\left(\dfrac{\sin\varphi}{\varphi}\right)^{\frac{1}{\varphi^2}}$. *Ans.* $\dfrac{1}{\sqrt[6]{e}}$.

52. $\lim\limits_{x\to 1}\left(\tan\dfrac{\pi x}{4}\right)^{\tan\frac{\pi x}{2}}$. *Ans.* $\dfrac{1}{e}$.