Lesson 17 (even problems must be solved in class, odd examples must be solved at home)

Equations of a Tangent and a Normal.
Lengths of a Subtangent and a Subnormal

207. Write the equations of the tangent and the normal to the curve $y=x^3-3x^2-x+5$ at the point $M(3, 2)$. *Ans.* The tangent is $8x-y-22=0$; the normal, $x+8y-19=0$.

208. Find the equations of the tangent and normal, the lengths of the subtangent and subnormal of the circle $x^2+y^2=r^2$ at the point $M(x_1, y_1)$. *Ans.* The tangent is $xx_1+yy_1=r^2$; the normal is $x_1y-y_1x=0$; $S_T=\left|\frac{y_1^2}{x_1}\right|$; $S_N=|x_1|$.

209. Show that the subtangent of the parabola $y^2=4px$ at any point is divided into two by the vertex, and the subnormal is constant and equal to $2p$. Make a drawing.

210. Find the equation of the tangent at the point $M(x_1, y_1)$

(a) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. *Ans.* $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$;

(b) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. *Ans.* $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$.

211. Find the equations of the tangent and normal to the Witch of Agnesi $y=\frac{8a^3}{4a^2+x^2}$ at the point where $x=2a$. *Ans.* The tangent is $x+2y=4a$; the normal is $y=2x-3a$.

212. Show that the normal to the curve $3y=6x-5x^3$ drawn to the point $M\left(1, \frac{1}{3}\right)$ passes through the coordinate origin.

213. Show that the tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point $M(a, b)$ is $\frac{x}{a}+\frac{y}{b}=2$.

214. Find the equation of that tangent to the parabola, $y^2=20x$, which forms an angle of 45° with the x-axis. *Ans.* $y=x+5$ [at the point (5, 10)].

215. Find the equations of those tangents to the circle $x^2+y^2=52$ which are parallel to the straight line $2x+3y=6$. *Ans.* $2x+3y\pm 26=0$.

216. Find the equations of those tangents to the hyperbola $4x^2-9y^2=36$ which are perpendicular to the straight line $2y+5x=10$. *Ans.* There are no such tangents.

217. Show that the segment (lying between the coordinate axes) of the tangent to the hyperbola $xy=m$ is divided into two by the point of tangency.

218. Prove that the segment (between the coordinate axes) of a tangent to the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is of constant length.

219. At what angle α do the curves $y=a^x$ and $y=b^x$ intersect? *Ans.* $\tan\alpha=\frac{\ln a-\ln b}{1+\ln a\cdot\ln b}$.

220. Find the lengths of the subtangent, subnormal, tangent and normal to the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ at the point at which $\theta=\frac{\pi}{2}$.

Ans. $S_T=a$, $S_N=a$, $T=a\sqrt{2}$, $N=a\sqrt{2}$.

221. Find the quantities S_T, S_N, T and N for the astroid $x=4a\cos^3 t$, $y=4a\sin^3 t$. *Ans.* $S_T=|4a\sin^2 t\ \cos t|$; $S_N=|4a\sin^3 t\ \tan t|$; $T=4a\sin^2 t$; $N=|4a\sin^2 t\ \tan t|$.