

Lesson 16 (even problems must be solved in class, odd examples must be solved at home)

186. $y = \cos ax$. Find $y^{(n)}$. Ans. $a^n \cos \left(ax + n \frac{\pi}{2} \right)$.
187. $y = a^x$. Find $y^{(n)}$. Ans. $(\ln a)^n a^x$. 188. $y = \ln(1+x)$. Find $y^{(n)}$. Ans. $(-1)^{n-1} \frac{(n-1)!}{(1+x)^n}$.
189. $y = \frac{1-x}{1+x}$. Find $y^{(n)}$. Ans. $2(-1)^n \frac{n!}{(1+x)^{n+1}}$. 190. $y = e^x x$. Find $y^{(n)}$. Ans. $e^x (x+n)$.
191. $y = x^{n-1} \ln x$. Find $y^{(n)}$. Ans. $\frac{(n-1)!}{x}$.
192. $y = \sin^2 x$. Find $y^{(n)}$. Ans. $-2^{n-1} \cos \left(2x + n \frac{\pi}{2} \right)$. 193. $y = x \sin x$. Find $y^{(n)}$. Ans. $x \sin \left(x + n \frac{\pi}{2} \right) - n \cos \left(x + n \frac{\pi}{2} \right)$.
194. If $y = e^x \sin x$, prove that $y'' - 2y' + 2y = 0$. 195. $y^2 = 4ax$. Find $\frac{d^2 y}{dx^2}$. Ans. $-\frac{4a^2}{y^3}$.
196. $b^2 x^2 + a^2 y^2 = a^2 b^2$. Find $\frac{d^2 y}{dx^2}$ and $\frac{d^3 y}{dx^3}$. Ans. $-\frac{b^4}{a^2 y^3}$; $-\frac{3b^6 x}{a^4 y^5}$.
197. $x^2 + y^2 = r^2$. Find $\frac{d^2 y}{dx^2}$. Ans. $-\frac{r^2}{y^3}$. 198. $y^2 - 2xy = 0$. Find $\frac{d^3 y}{dx^3}$. Ans. 0.
199. $\rho = \tan(\varphi + \rho)$. Find $\frac{d^3 \rho}{d\varphi^3}$. Ans. $-\frac{2(5 + 8\rho^2 + 3\rho^4)}{\rho^5}$.
200. $\sec \varphi \cos \rho = C$. Find $\frac{d^2 \rho}{d\varphi^2}$. Ans. $\frac{\tan^2 \rho - \tan^2 \varphi}{\tan^3 \rho}$.
201. $e^x + x = e^y + y$. Find $\frac{d^2 y}{dx^2}$. Ans. $\frac{(1 - e^{x+y})(e^x - e^y)}{(e^y + 1)^3}$.
202. $y^3 + x^3 - 3axy = 0$. Find $\frac{d^2 y}{dx^2}$. Ans. $-\frac{2a^3 xy}{(y^3 - ax)^3}$.
203. $x = a(t - \sin t)$, $y = a(1 - \cos t)$. Find $\frac{d^2 y}{dx^2}$. Ans. $-\frac{1}{4a \sin^4 \left(\frac{t}{2} \right)}$.
204. $x = a \cos 2t$, $y = b \sin^2 t$. Show that $\frac{d^2 y}{dx^2} = 0$.
205. $x = a \cos t$, $y = a \sin t$. Find $\frac{d^3 y}{dx^3}$. Ans. $-\frac{3 \cos t}{a^2 \sin^5 t}$.
206. Show that $\frac{d^{2n}}{dx^{2n}} (\sinh x) = \sinh x$; $\frac{d^{2n+1}}{dx^{2n+1}} (\sinh x) = \cosh x$.