Lesson 15 (even problems must be solved in class, odd examples must be solved at home)

Find the differentials of the following functions:

162. $y = (a^2 - x^2)^5$. Ans. $dy = -10x(a^2 - x^2)^4 dx$.   163. $y = \sqrt{1 + x^2}$.   Ans.

$dy = \dfrac{x\,dx}{\sqrt{1 + x^2}}$.

164. $y = \dfrac{1}{3}\tan^3 x + \tan x$.   Ans. $dy = \sec^4 x\,dx$.

165. $y = \dfrac{x\ln x}{1 - x} + \ln(1 - x)$. Ans. $dy = \dfrac{\ln x\,dx}{(1 - x)^2}$.

Calculate the increments and differentials of the following functions:

166. $y = 2x^2 - x$ when $x = 1$, $\Delta x = 0.01$. Ans. $\Delta y = 0.0302$, $dy = 0.03$. 167. Given y $x^3 + 2x$. Find Δy and dy when $x = -1$, $\Delta x = 0.02$. Ans. $\Delta y = 0.098808$, dy -0.1. 168. Given $y = \sin x$. Find dy when $x = \dfrac{\pi}{3}$, $\Delta x = \dfrac{\pi}{18}$. Ans. $dy = \dfrac{\pi}{36} =$

-0.0873. 169. Knowing that $\sin 60° = \dfrac{\sqrt{3}}{2} = 0.866025$, $\cos 60° = \dfrac{1}{2}$, find the approximate values of $\sin 60°3'$ and $\sin 60°18'$. Compare the results with tabular data. Ans. $\sin 60°3' \approx 0.866461$, $\sin 60°18' \approx 0.868643$. 170. Find the approximate value of $\tan 45°4'30''$. Ans. 1.00262. 171. Knowing that $\log_{10} 200 = 2.30103$ find the approximate value of $\log_{10} 200.2$. Ans. 2.30146.

Derivatives of different orders. 172. $y = 3x^3 - 2x^2 + 5x - 1$. Find y''. Ans. $18x - 4$. 173. $y = \sqrt[5]{x^3}$. Find y'''. Ans. $\dfrac{42}{125}x^{-\frac{12}{5}}$. 174. $y = x^6$. Find $y^{(6)}$. Ans. $6!$. 175. $y = \dfrac{C}{x^n}$. Find y''. Ans. $\dfrac{n(n+1)C}{x^{n+2}}$. 176. $y = \sqrt{a^2 - x^2}$. Find y''. Ans.

$-\dfrac{a^2}{(a^2 - x^2)\sqrt{a^2 - x^2}}$. 177. $y = 2\sqrt{x}$. Find $y^{(4)}$. Ans. $-\dfrac{15}{8\sqrt{x^7}}$. 178. $y =$ $ax^3 + bx + c$. Find y'''. Ans. 0. 179. $f(x) = \ln(x+1)$. Find $f^{(4)}(x)$. Ans. $-\dfrac{6}{(x+1)^4}$. 180. $y = \tan x$. Find y'''. Ans. $6\sec^4 x - 4\sec^2 x$. 181. $y = \ln\sin x$. Find y'''. Ans. $2\cot x \csc^2 x$. 182. $f(x) = \sqrt{\sec 2x}$. Find $f''(x)$. Ans. $f''(x) = -3[f(x)]^5 - f(x)$. 183. $y = \dfrac{x^3}{1 - x}$. Find $f^{(4)}(x)$. Ans. $\dfrac{4!}{(1 - x)^5}$. 184. $p =$ $(q^2 + a^2)\arctan\dfrac{q}{a}$. Find $\dfrac{d^3 p}{dq^3}$. Ans. $\dfrac{4a^3}{(a^2 + q^2)^2}$. 185. $y = \dfrac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$. Find $\dfrac{d^2 y}{dx^2}$. Ans. $\dfrac{y}{a^2}$.