

Lesson 14 (even problems must be solved in class, odd examples must be solved at home)

Differentiation of Implicit Functions

- Find $\frac{dy}{dx}$ if:
142. $y^2 = 4px$. Ans. $\frac{dy}{dx} = \frac{2p}{y}$.
 143. $x^2 + y^2 = a^2$. Ans. $\frac{dy}{dx} = -\frac{x}{y}$.
 144. $b^2x^2 + a^2y^2 = a^2b^2$. Ans. $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$.
 145. $y^3 - 3y + 2ax = 0$. Ans. $\frac{dy}{dx} = \frac{2a}{3(1-y^2)}$.
 146. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = a^{\frac{1}{2}}$. Ans. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$.
 147. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. Ans. $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$.
 148. $y^2 - 2xy + b^2 = 0$. Ans. $\frac{dy}{dx} = \frac{y}{y-x}$.
 149. $x^3 + y^3 - 3axy = 0$. Ans. $\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$.
 150. $y = \cos(x+y)$. Ans. $\frac{dy}{dx} = -\frac{\sin(x+y)}{1+\sin(x+y)}$.
 151. $\cos(xy) = x$. Ans. $\frac{dy}{dx} = -\frac{1+y\sin(xy)}{x\sin(xy)}$.

Find $\frac{dy}{dx}$ of the following functions represented parametrically:

152. $x = a \cos t$, $y = b \sin t$. Ans. $\frac{dy}{dx} = -\frac{b}{a} \cot t$.
153. $x = a(t - \sin t)$, $y = a(1 - \cos t)$. Ans. $\frac{dy}{dx} = \cot \frac{t}{2}$.
154. $x = a \cos^3 t$, $y = b \sin^3 t$. Ans. $\frac{dy}{dx} = -\frac{b}{a} \tan t$.
155. $x = \frac{3at}{1+t^2}$, $y = \frac{3at^2}{1+t^2}$. Ans. $\frac{dy}{dx} = \frac{2t}{1-t^2}$.
156. $u = 2 \ln \cot s$, $v = \tan s + \cot s$.

Show that $\frac{du}{dv} = \tan 2s$.

Find the tangents of the angles of inclination of tangent lines to curves:

157. $x = \cos t$, $y = \sin t$ at the point $x = -\frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$. Make a drawing.
- Ans. $\frac{1}{\sqrt{3}}$.
158. $x = 2 \cos t$, $y = \sin t$ at the point $x = 1$, $y = -\frac{\sqrt{3}}{2}$. Make a drawing.
- Ans. $\frac{1}{2\sqrt{3}}$.
159. $x = a(t - \sin t)$, $y = a(1 - \cos t)$ when $t = \frac{\pi}{2}$.

Make a drawing. Ans. 1.

160. $x = a \cos^3 t$, $y = a \sin^3 t$ when $t = \frac{\pi}{4}$. Make a drawing.
- Ans. —1.
161. A body thrown at an angle α to the horizon (in airless space) described a curve parabola, under the force of gravity, whose equations are: $x = (v_0 \cos \alpha)t$, $y = (v_0 \sin \alpha)t - \frac{gt^2}{2}$ ($g = 9.8 \text{ m/sec}^2$). Knowing that $\alpha = 60^\circ$, $v_0 = 50 \text{ m/sec}$, determine the direction of motion when: (1) $t = 2 \text{ sec}$, (2) $t = 7 \text{ sec}$. Make a drawing.
- Ans. (1) $\tan \varphi_1 = 0.948$, $\varphi_1 = 43^\circ 30'$; (2) $\tan \varphi_2 = -1.012$, $\varphi_2 = +134^\circ 3'$.