Lesson 13 (even problems must be solved in class, odd examples must be solved at home)

Find the derivatives of the following functions after first taking logarithms of the functions:

116. $y = \arcsin \dfrac{x}{a}$.

Ans. $y' = \dfrac{1}{\sqrt{a^2 - x^2}}$.

117. $y = (\arcsin x)^2$.

Ans. $y' = \dfrac{2 \arcsin x}{\sqrt{1 - x^2}}$.

118. $y = \arctan(x^2 + 1)$. *Ans.* $y' = \dfrac{2x}{1 + (x^2 + 1)^2}$.

119. $y = \arctan \dfrac{2x}{1 - x^2}$.

Ans. $y' = \dfrac{2}{1 + x^2}$. **120.** $y = \arccos(x^2)$. *Ans.* $y' = \dfrac{-2x}{\sqrt{1 - x^4}}$. **121.** $y = \dfrac{\arccos x}{x}$.

Ans. $y' = \dfrac{-(x + \sqrt{1 - x^2}\,\arccos x)}{x^2 \sqrt{1 - x^2}}$. **122.** $y = \arcsin \dfrac{x + 1}{\sqrt{2}}$. *Ans.* $y' =$

$\dfrac{1}{\sqrt{1 - 2x - x^2}}$ **123.** $y = x\sqrt{a^2 - x^2} + a^2 \arcsin \dfrac{x}{a}$. *Ans.* $y' = 2\sqrt{a^2 - x^2}$.

124. $y = \sqrt{a^2 - x^2} + a \arcsin \dfrac{x}{a}$. *Ans.* $y' = \sqrt{\dfrac{a - x}{a + x}}$. **125.** $u = \arctan \dfrac{v + a}{1 - av}$.

Ans. $\dfrac{du}{dv} = \dfrac{1}{1 + v^2}$. **126.** $y = \dfrac{1}{\sqrt{3}} \arctan \dfrac{x\sqrt{3}}{1 - x^2}$. *Ans.* $y' = \dfrac{x^2 + 1}{x^4 + x^2 + 1}$. **127.**

$y = x \arcsin x$. *Ans.* $y' = \arcsin x + \dfrac{x}{\sqrt{1 - x^2}}$. **128.** $f(x) = \arccos(\ln x)$. *Ans.*

$f'(x) = -\dfrac{1}{x\sqrt{1 - \ln^2 x}}$. **129.** $f(x) = \arcsin \sqrt{\sin x}$. *Ans.* $f'(x) = \dfrac{\cos x}{2\sqrt{\sin x - \sin^2 x}}$.

130. $y = \arctan \sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$ $(0 \leqslant x < \pi)$. *Ans.* $y' = \dfrac{1}{2}$. **131.** $y = e^{\arctan x}$.

Ans. $y' = \dfrac{e^{\arctan x}}{1 + x^2}$. **132.** $y = \arctan \dfrac{e^x - e^{-x}}{2}$. *Ans.* $y' = \dfrac{2}{e^x + e^{-x}}$. **133.** $y =$

$x^{\arcsin x}$. *Ans.* $y' = x^{\arcsin x}\left(\dfrac{\arcsin x}{x} + \dfrac{\ln x}{\sqrt{1 - x^2}}\right)$. **134.** $y = \arcsin(\sin x)$.

Ans. $y' = \dfrac{\cos x}{|\cos x|} = \begin{cases} +1 & \text{in 1st and 4th quadrants.} \\ -1 & \text{in 2nd and 3rd quadrants.} \end{cases}$ **135.** $y = \arctan \dfrac{4 \sin x}{3 + 5 \cos x}$.

Ans. $y' = \dfrac{4}{5 + 3 \cos x}$. **136.** $y = \arctan \dfrac{a}{x} + \ln \sqrt{\dfrac{x - a}{x + a}}$. *Ans.* $y' = \dfrac{2a^3}{x^4 - a^4}$.

137. $y = \ln\left(\dfrac{1 + x}{1 - x}\right)^{\frac{1}{4}} - \dfrac{1}{2} \arctan x$. *Ans.* $y' = \dfrac{x^2}{1 - x^4}$. **138.** $y = \dfrac{3x^2 - 1}{3x^3} +$

$+ \ln \sqrt{1 + x^2} + \arctan x$. *Ans.* $y' = \dfrac{x^5 + 1}{x^6 + x^4}$. **139.** $y = \dfrac{1}{3} \ln \dfrac{x + 1}{\sqrt{x^2 - x + 1}} +$

$+ \dfrac{1}{\sqrt{3}} \arctan \dfrac{2x - 1}{\sqrt{3}}$. *Ans.* $y' = \dfrac{1}{x^3 + 1}$. **140.** $y = \ln \dfrac{1 + x\sqrt{2} + x^2}{1 - x\sqrt{2} + x^2} +$

$+ 2 \arctan \dfrac{x\sqrt{2}}{1 - x^2}$. *Ans.* $y' = \dfrac{4\sqrt{2}}{1 + x^4}$. **141.** $y = \arccos \dfrac{x^{2n} - 1}{x^{2n} + 1}$. *Ans.* $-\dfrac{2n|x|^n}{x(x^{2n} + 1)}$.