Lesson 12 (even problems must be solved in class, odd examples must be solved at home)

Find the derivatives of the following functions:

99. $y=x^x$. *Ans.* $y'=x^x(\ln x+1)$. **100.** $y=x^{\frac{1}{x}}$. *Ans.* $y'=x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$.
101. $y=x^{\ln x}$. *Ans.* $y'=x^{\ln x-1}\ln x^2$. **102.** $y=e^{x^x}$. *Ans.* $y'=e^{x^x}(1+\ln x)x^x$.
103. $y=\left(\frac{x}{n}\right)^{nx}$ *Ans.* $y'=n\left(\frac{x}{n}\right)^{nx}\left(1+\ln\frac{x}{n}\right)$. **104.** $y=x^{\sin x}$.
Ans. $y'=x^{\sin x}\left(\frac{\sin x}{x}+\ln x\cos x\right)$. **105.** $y=(\sin x)^x$. *Ans.* $y'=(\sin x)^x\times$
$\times(\ln\sin x+x\cot x)$. **106.** $y=(\sin x)^{\tan x}$. *Ans.* $y'=(\sin x)^{\tan x}(1+\sec^2 x\ln\sin x)$.
107. $y=\tan\frac{1-e^x}{1+e^x}$. *Ans.* $y'=-\frac{2e^x}{(1+e^x)^2}\frac{1}{\cos^2\frac{1-e^x}{1+e^x}}$. **108.** $y=\sin\sqrt{1-2^x}$.
Ans. $y'=-\frac{\cos\sqrt{1-2^x}}{2\sqrt{1-2^x}}2^x\ln 2$. **109.** $y=10^{x\tan x}$. *Ans.* $y'=10^{x\tan x}\ln 10\times$
$\times\left(\tan x+\frac{x}{\cos^2 x}\right)$.

Find the derivatives of the following functions after first taking logarithms of the functions:

110. $y=\sqrt[3]{\frac{x(x^2+1)}{(x-1)^2}}$. *Ans.* $y'=\frac{1}{3}\sqrt[3]{\frac{x(x^2+1)}{(x-1)^2}}\left(\frac{1}{x}+\frac{2x}{x^2+1}+\frac{2}{x-1}\right)$.

111. $y=\frac{(x+1)^3\sqrt[4]{(x-2)^3}}{\sqrt[5]{(x-3)^2}}$. *Ans.* $y'=\frac{(x+1)^3\sqrt[4]{(x-2)^3}}{\sqrt[5]{(x-3)^2}}\left(\frac{3}{x+1}+\frac{3}{4(x-2)}-\right.$
$\left.-\frac{2}{5(x-3)}\right)$. **112.** $y=\frac{(x+1)^2}{(x+2)^3(x+3)^4}$. *Ans.* $y'=-\frac{(x+1)(5x^2+14x+5)}{(x+2)^4(x+3)^5}$.
113. $y=\frac{\sqrt[5]{(x-1)^2}}{\sqrt[4]{(x-2)^3}\sqrt[3]{(x-3)^7}}$. *Ans.* $y'=\frac{-161x^2+480x-271}{60\sqrt[5]{(x-1)^3}\sqrt[4]{(x-2)^7}\sqrt[3]{(x-3)^{10}}}$.
114. $y=\frac{x(1+x^2)}{\sqrt{1-x^2}}$. *Ans.* $y'=\frac{1+3x^2-2x^4}{(1-x^2)^{\frac{3}{2}}}$. **115.** $y=x^5(a+3x)^3(a-2x)^2$.

Ans. $y'=5x^4(a+3x)^2(a-2x)(a^2+2ax-12x^2)$.