

# Lesson 11 (even problems must be solved in class, odd examples must be solved at home)

Find the derivatives.

51.  $y = \frac{1}{2} \tan^2 x$ . Ans.  $y' = \tan x \sec^2 x$ . 52.  $y = \ln \cos x$ . Ans.  $y' = -\tan x$ .

53.  $y = \ln \tan x$ . Ans.  $y' = \frac{2}{\sin 2x}$ . 54.  $y = \ln \sin^2 x$ . Ans.  $y' = 2 \cot x$ .

55.  $y = \frac{\tan x - 1}{\sec x}$ . Ans.  $y' = \sin x + \cos x$ . 56.  $y = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}}$ .

Ans.  $y' = \frac{1}{\cos x}$ . 57.  $y = \ln \tan \left( \frac{\pi}{4} + \frac{x}{2} \right)$ . Ans.  $y' = \frac{1}{\cos x}$ .

58.  $y = \sin(x+a) \cos(x+a)$ . Ans.  $y' = \cos 2(x+a)$ . 59.  $f(x) = \sin(\ln x)$ .

Ans.  $f'(x) = \frac{\cos(\ln x)}{x}$ . 60.  $f(x) = \tan(\ln x)$ . Ans.  $f'(x) = \frac{\sec^2(\ln x)}{x}$ .

61.  $f(x) = \sin(\cos x)$ . Ans.  $f'(x) = -\sin x \cos(\cos x)$ . 62.  $r = \frac{1}{3} \tan^3 \phi - \tan \phi + \phi$ .

Ans.  $\frac{dr}{d\phi} = \tan^4 \phi$ . 63.  $f(x) = (x \cot x)^2$ . Ans.  $f'(x) = 2x \cot x (\cot x - x \csc^2 x)$ .

64.  $y = \ln(ax+b)$ . Ans.  $y' = \frac{a}{ax+b}$ . 65.  $y = \log_a(x^2+1)$ . Ans.  $y' = \frac{2x}{(x^2+1) \ln a}$ .

66.  $y = \ln \frac{1+x}{1-x}$ . Ans.  $y' = \frac{2}{1-x^2}$ . 67.  $y = \log_3(x^2 - \sin x)$ .

Ans.  $y' = \frac{2x - \cos x}{(x^2 - \sin x) \ln 3}$ . 68.  $y = \ln \frac{1+x^2}{1-x^2}$ . Ans.  $y' = \frac{4x}{1-x^4}$ .

69.  $y = \ln(x^2+x)$ . Ans.  $y' = \frac{2x+1}{x^2+x}$ . 70.  $y = \ln(x^3-2x+5)$ .

Ans.  $y' = \frac{3x^2-2}{x^3-2x+5}$ . 71.  $y = x \ln x$ . Ans.  $y' = \ln x + 1$ . 72.  $y = \ln^3 x$ .

Ans.  $y' = \frac{3 \ln^2 x}{x}$ . 73.  $y = \ln(x + \sqrt{1+x^2})$ . Ans.  $y' = \frac{1}{\sqrt{1+x^2}}$ .

74.  $y = \ln(\ln x)$ . Ans.  $y' = \frac{1}{x \ln x}$ . 75.  $f(x) = \ln \sqrt{\frac{1+x}{1-x}}$ . Ans.  $f'(x) = \frac{1}{1-x^2}$ .

76.  $f(x) = \ln \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x}$ . Ans.  $f'(x) = -\frac{2}{\sqrt{1+x^2}}$ . 77.  $y = \sqrt{a^2+x^2} -$

$-a \ln \frac{a + \sqrt{a^2+x^2}}{x}$ . Ans.  $y' = \frac{\sqrt{a^2+x^2}}{x}$ . 78.  $y = \ln(x + \sqrt{x^2+a^2}) -$

$-\frac{\sqrt{x^2+a^2}}{x}$ . Ans.  $y' = \frac{\sqrt{x^2+a^2}}{x^2}$ . 79.  $y = -\frac{\cos x}{2 \sin^2 x} + \frac{1}{2} \ln \tan \frac{x}{2}$ . Ans.

$y' = \frac{1}{\sin^3 x}$ . 80.  $y = \frac{\sin x}{2 \cos^2 x}$ . Ans.  $y' = \frac{1 + \sin^2 x}{2 \cos^3 x}$ . 81.  $y = \frac{1}{2} \tan^2 x + \ln \cos x$ .

Ans.  $y' = \tan^3 x$ . 82.  $y = e^{ax}$ . Ans.  $y' = ae^{ax}$ . 83.  $y = e^{4x+5}$ . Ans.  $y' = 4e^{4x+5}$ .

84.  $y = a^{x^2}$ . Ans.  $y' = 2xa^{x^2} \ln a$ . 85.  $y = 7^{x^2+2x}$ . Ans.  $y' = 2(x+1)7^{x^2+2x} \ln 7$ .

86.  $y = c^{a^2-x^2}$ . Ans.  $y' = -2xc^{a^2-x^2} \ln c$ . 87.  $y = ae^{\sqrt{x}}$ . Ans.  $y' = \frac{a}{2\sqrt{x}} e^{\sqrt{x}}$ .

88.  $r = a^\theta$ . Ans.  $r' = a^\theta \ln a$ . 89.  $r = a^{\ln \theta}$ . Ans.  $\frac{dr}{d\theta} = \frac{a^{\ln \theta} \ln a}{\theta}$ . 90.  $y = e^x (1-x^2)$ .

Ans.  $y' = e^x (1-2x-x^2)$ . 91.  $y = \frac{e^x-1}{e^x+1}$ . Ans.  $y' = \frac{2e^x}{(e^x+1)^2}$ . 92.  $y = \ln \frac{e^x}{1+e^x}$ .

$Ans. y' = \frac{1}{1+e^x}.$       93.  $y = \frac{a}{2} \left( e^{\frac{x}{a}} - e^{-\frac{x}{a}} \right).$        $Ans. y' = \frac{1}{2} \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right).$   
 94.  $y = e^{\sin x}.$        $Ans. y' = e^{\sin x} \cos x.$       95.  $y = a^{\tan nx}.$        $Ans. y' = na^{\tan nx} \sec^2 nx \ln a.$   
 96.  $y = e^{\cos x} \sin x.$        $Ans. y' = e^{\cos x} (\cos x - \sin^2 x).$       97.  $y = e^x \ln \sin x.$   
 $Ans. y' = e^x (\cot x + \ln \sin x).$       98.  $y = x^n e^{\sin x}.$        $Ans. y' = x^{n-1} e^{\sin x} (n + x \cos x).$