Lesson 10

Find the derivatives of the following functions using the definition of a derivative:

1. $y=x^2$. *Ans.* $3x^2$. 2. $y=\frac{1}{x}$. *Ans.* $-\frac{1}{x^2}$. 3. $y=\sqrt{x}$. *Ans.* $\frac{1}{2\sqrt{x}}$. 4. $y=\frac{1}{\sqrt{x}}$. *Ans.* $-\frac{1}{2x\sqrt{x}}$. 5. $y=\sin^2 x$. *Ans.* $2\sin x\cos x$. 6. $y=2x^2-x$. *Ans.* $4x-1$.

Determine the tangents of the angles of inclination of the tangent line to the curves:

7. $y=x^3$. (a) When $x=1$. *Ans.* 3. (b) When $x=-1$. *Ans.* 3. Make a drawing. 8. $y=\frac{1}{x}$. (a) When $x=\frac{1}{2}$. *Ans.* -4. (b) When $x=1$. *Ans.* -1. Make a drawing. 9. $y=\sqrt{x}$ when $x=2$. *Ans.* $\frac{1}{2\sqrt{2}}$.

Find the derivatives of the following functions:

10. $y=x^4+3x^2-6$. *Ans.* $y'=4x^3+6x$. 11. $y=6x^3-x^2$. *Ans.* $y'=18x^2-2x$. 12. $y=\frac{x^5}{a+b}-\frac{x^2}{a-b}-x$. *Ans.* $y'=\frac{5x^4}{a+b}-\frac{2x}{a-b}-1$. 13. $y=\frac{x^3-x^2+1}{5}$. *Ans.* $y'=\frac{3x^2-2x}{5}$. 14. $y=2ax^3-\frac{x^2}{b}+c$. *Ans.* $y'=6ax^2-\frac{2x}{b}$. 15. $y=6x^{\frac{7}{2}}+4x^{\frac{5}{2}}+2x$. *Ans.* $y'=21x^{\frac{5}{2}}+10x^{\frac{3}{2}}+2$. 16. $y=\sqrt{3x}+\sqrt[3]{x}+\frac{1}{x}$. *Ans.* $y'=\frac{\sqrt{3}}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}-\frac{1}{x^2}$ 17. $y=\frac{(x+1)^3}{x^{\frac{3}{2}}}$. *Ans.* $y'=\frac{3(x+1)^2(x-1)}{2x^{\frac{5}{2}}}$. 18. $y=\frac{x}{m}+\frac{m}{x}+\frac{x^2}{n^2}+\frac{n^2}{x^2}$. *Ans.* $y'=\frac{1}{m}-\frac{m}{x^2}+\frac{2x}{n^2}-\frac{2n^2}{x^3}$. 19. $y=\sqrt[3]{x^2}-2\sqrt{x}+5$. *Ans.* $y'=\frac{2}{3}\frac{1}{\sqrt[3]{x}}-\frac{1}{\sqrt{x}}$. 20. $y=\frac{ax^2}{\sqrt[3]{x}}+\frac{b}{x\sqrt{x}}-\frac{\sqrt[3]{x}}{\sqrt{x}}$. *Ans.* $y'=\frac{5}{3}ax^{\frac{2}{3}}-\frac{3}{2}bx^{-\frac{5}{2}}+\frac{1}{6}x^{-\frac{7}{6}}$. 21. $y=(1+4x^3)(1+2x^2)$. *Ans.* $y'=4x(1+3x+10x^3)$. 22. $y=x(2x-1)(3x+2)$. *Ans.* $y'=2(9x^2+x-1)$. 23. $y=(2x-1)(x^2-6x+3)$. *Ans.* $y'=6x^2-26x+12$. 24. $y=\frac{2x^4}{b^2-x^2}$. *Ans.* $y'=\frac{4x^3(2b^2-x^2)}{(b^2-x^2)^2}$. 25. $y=\frac{a-x}{a+x}$. *Ans.* $y'=-\frac{2a}{(a+x)^2}$. 26. $f(t)=\frac{t^3}{1+t^2}$. *Ans.* $f'(t)=\frac{t^2(3+t^2)}{(1+t^2)^2}$. 27. $f(s)=\frac{(s+4)^2}{s+3}$. *Ans.* $f'(s)=\frac{(s+2)(s+4)}{(s+3)^2}$. 28. $y=\frac{x^3+2}{x^2-x-2}$. *Ans.* $y'=\frac{x^4-2x^3-6x^2-4x+2}{(x^2-x-2)^2}$. 29. $y=\frac{x^p}{x^m-a^m}$. *Ans.* $y'=\frac{x^{p-1}[(p-m)x^m-pa^m]}{(x^m-a^m)^2}$. 30. $y=(2x^2-3)^2$. *Ans.* $y'=8x(2x^2-3)$. 31. $y=(x^2+a^2)^5$. *Ans.* $y=10x(x^2+a^2)^4$. 32. $y=\sqrt{x^2+a^2}$.

Ans. $y'=\frac{x}{\sqrt{x^2+a^2}}$. **33.** $y=(a+x)\sqrt{a-x}$. *Ans.* $y'=\frac{a-3x}{2\sqrt{a-x}}$. **34.** $y=\sqrt{\frac{1+x}{1-x}}$. *Ans.* $y'=\frac{1}{(1-x)\sqrt{1-x^2}}$. **35.** $y=\frac{2x^2-1}{x\sqrt{1+x^2}}$. *Ans.* $y'=-\frac{1+4x^2}{x^2(1+x^2)^{\frac{3}{2}}}$. **36.** $y=\sqrt[3]{x^2+x+1}$. *Ans.* $y'=\frac{2x+1}{3\sqrt[3]{(x^2+x+1)^2}}$. **37.** $y=-(1+\sqrt[3]{x})^3$. *Ans.* $y'=\left(1+\frac{1}{\sqrt[3]{x}}\right)^2$. **38.** $y=\sqrt{x+\sqrt{x+\sqrt{x}}}$. *Ans.* $y'=\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right]$. **39.** $y=\sin^2 x$. *Ans.* $y'=\sin 2x$. **40.** $y=2\sin x+\cos 3x$. *Ans.* $y'=2\cos x-3\sin 3x$. **41.** $y=\tan(ax+b)$. *Ans.* $y'=\frac{a}{\cos^2(ax+b)}$. **42.** $y=\frac{\sin x}{1+\cos x}$. *Ans.* $y'=\frac{1}{1+\cos x}$. **43.** $y=\sin 2x\cos 3x$. *Ans.* $y'=2\cos 2x\cos 3x-3\sin 2x\sin 3x$. **44.** $y=\cot^2 5x$. *Ans.* $y'=-10\cot 5x\csc^2 5x$. **45.** $y=t\sin t+\cos t$. *Ans.* $y'=t\cos t$. **46.** $y=\sin^3 t\cos t$. *Ans.* $y'=\sin^2 t(3\cos^2 t-\sin^2 t)$. **47.** $y=a\sqrt{\cos 2x}$. *Ans.* $y'=-\frac{a\sin 2x}{\sqrt{\cos 2x}}$. **48.** $r=a\sin^3\frac{\varphi}{3}$. *Ans.* $r'_\varphi=a\sin^2\frac{\varphi}{3}\cos\frac{\varphi}{3}$. **49.** $y=\frac{\tan\frac{x}{2}+\cot\frac{x}{2}}{x}$. *Ans.* $y'=-\frac{2(x\cos x+\sin x)}{x^2\sin^2 x}$. **50.** $y=a\sin^4\frac{x}{2}$. *Ans.* $y'=2a\sin^3\frac{x}{2}\cos\frac{x}{2}$.

Even examples must be solved in class, odd examples must be solved at home