Lecture 4

Graph of Functions set Implicitly

Suppose the equation $F(x, y) = 0$ is given.If the set of points on a plane XOY whose coordinates satisfy this equation consists of a finite number of continuous curves, each of which is a graph of an unambiguous function $y = y(x)$, then this equation is said to implicitly define the corresponding family of functions $y_1(x), y_2(x), \dots, y_k(x)$. If a point (x_0, y_0) lies on only one of these curves, then the condition $y(x_0) = y_0$ allows you to unambiguously select this curve from the entire family, i.e. the equation $F(x, y) = 0$ and the condition $y(x_0) = y_0$ define (or specify) an unambiguous implicit continuous function in the neighborhood of the point (x_0, y_0) such that $F(x, y(x)) \equiv 0$, $y(x_0) = y_0$.

The simplest equation of the form $F(x, y) = 0$ is the equation $x - f\ (\mathrm{y}) = 0$, which defines the function that is the inverse of f: $y = f^{-1}(x)$. The OY-axis swaps with the OX-axis when the plane XOY is symmetrically displayed relative to the bisector of the first coordinate angle. Thus, the curve $y = f(x)$ is symmetrical to the curve $x = f(y)$ or $y = f^{-1}(x)$ relative to this bisector. In this mapping, the continuous monotone function will be transformed into a continuous monotone function, i.e., the inverse function is unambiguous, continuous, and monotonic. If the continuous function $x = f(y)$ is not monotonous, then the curve defined by the equation $x - f\ (\mathrm{y}) = 0$ will no longer be the graph of the function $y = y(x)$, since there is no one-to-one dependence of the function on the argument.

If an equation $F(x, y) = 0$ can be solved with respect to one of the variables, then the construction of a set of points (x, y) for which this equation is valid follows from the previous discussions. Sometimes you can enter a parameter t so that the equation $F(x, y) = 0$ is equal to $\{x = x(t), y = y(t), t \in T\}$ (or more of these relations).

Example 1. Let's draw a sketch of the curve defined by the equation $x = y - \sin y$ in the system XOY.

Solution We have that $x(k\pi) = k\pi$, $k \in \mathbb{Z}$, $x(y)$ is a monotonic odd function. A sketch of the curve in the system YOX is shown in Fig. 25, a, and a sketch of the curve defined by the equation $x = y - \sin y$ (i.e., in the system XOY) is shown in Fig. 25, 6.

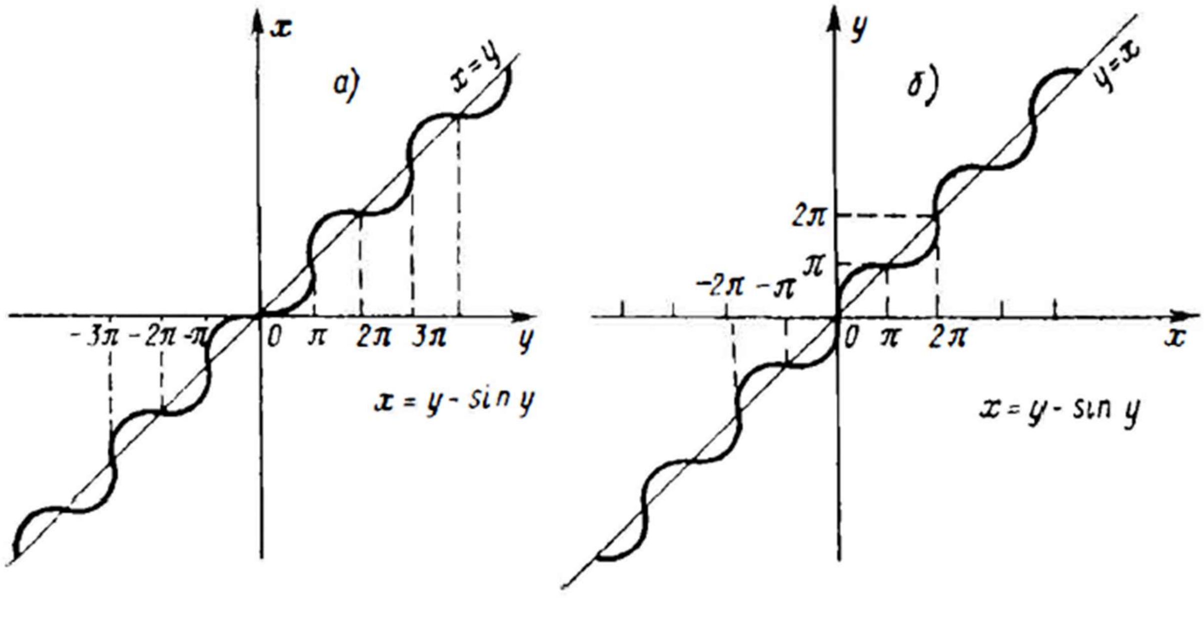

Fig. 25

Example 2. Let's draw a sketch of the curve defined by the equation $x = y \cdot \cos y$.

Solution: The function $x(y)$ is odd; for $y \geq 0$ have $|x| \leq y$, $x\left(\frac{\pi}{2} + k\pi\right) = 0$, $x(2k\pi) = 2k\pi$, $x\big((2k+1)\pi\big) = -(2k+1)\pi$. A sketch of the curve $x(y)$ in the system YOX is shown in Fig. 26, a, and the sketch of the curve $y(x)$ in the system XOY defined by the equation $x = y \cdot \cos y$ is shown in Fig. 26, 6.

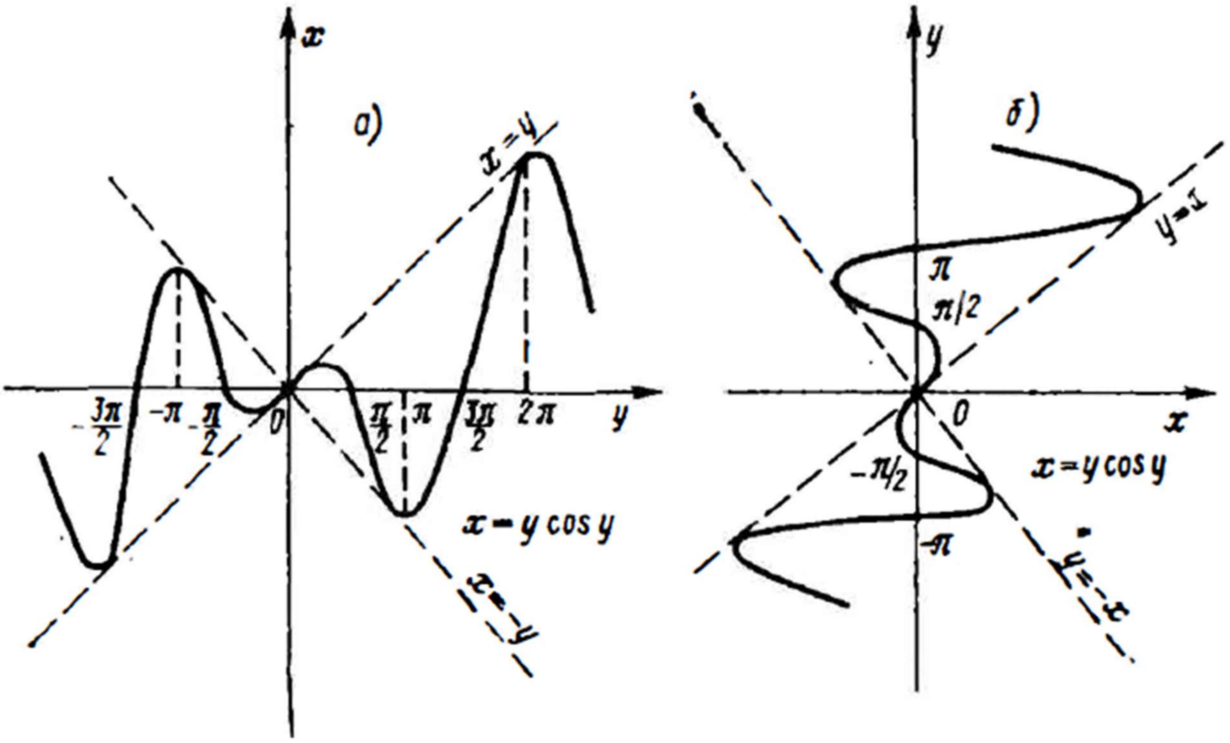

Fig. 26

Example H. Let's draw a sketch of the curve defined by the equation $x^5 + y^5 = 2x^2y^2$ in the XOY system.

Solution: Note that the point $(0{,}0)$ belongs to this curve. There are no other points of view $(0, y)$ on this curve. To construct the curve, enter the parameter $t = \frac{y}{x}$. This equation is then transformed as follows:

$$x^5(1 + t^5) = 2t^2x^4.$$

From this it can be seen that the equation $x^5 + y^5 = 2x^2y^2$ is equivalent to the relations

$$x(t) = 2t^2/(1 + t^5\ ), y(t) = 2t^3/(1 + t^5\ ),$$

since the point $(0{,}0)$ also belongs to this curve at $t = 0$. The construction of such curves was shown in the previous lecture. The curve is shown in Fig. 27.

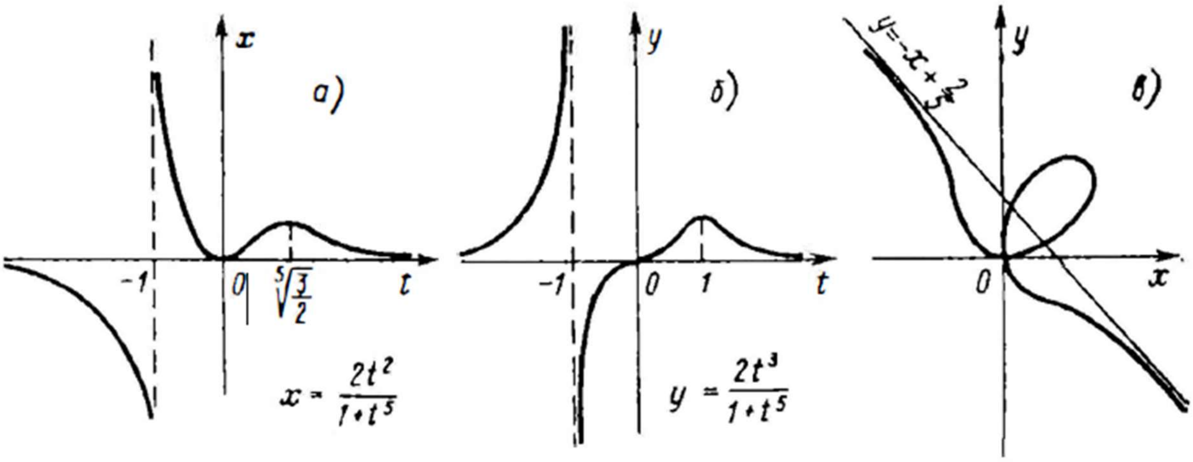

Fig. 27

Example 4. Let's draw a sketch of the curve defined by the equation $x^4 + y^4 = x^2 + y^2$. in the XOY system

Solution: Let's move on to the polar coordinate system combined with the Cartesian coordinate system, assuming $x = r \cdot \cos\varphi$, $y = r \cdot \sin\varphi$. Then the equation of this curve will take the form

$$r^2 = \frac{1}{\cos^4\varphi + \sin^4\varphi}, r = 0,$$

that is

$$r^2 = \frac{4}{3 + \cos 4\varphi}, r = 0.$$

Constructing curves like this one was done in the fourth practical lesson. A sketch of this curve is shown in Fig. 28.

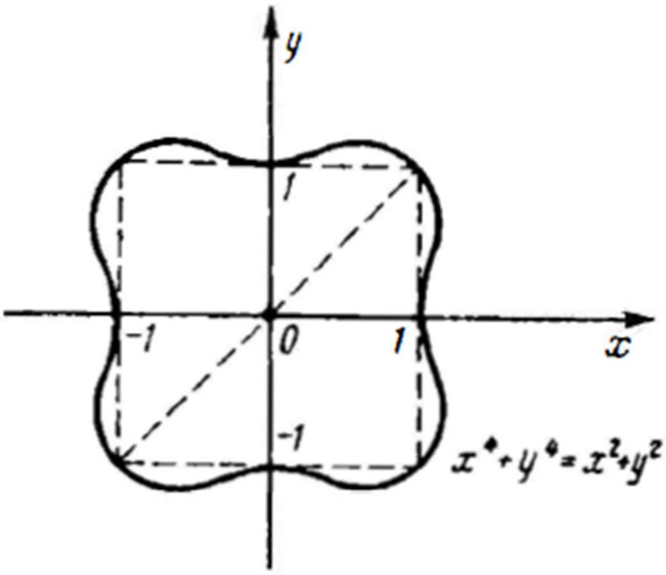

Fig. 28